U.G. Core Course – C8 (SET-1)

Semester – IV

Subject – Mathematics

F.M – 75

ANALYSIS – II

Group – A (Compulsory)

Q (1.) $1 \times 10 = 10$

(a.) Define Beta function.

(b.) Define Gamma function.

(c.) Transform $\iint v \, dx \, dy$ into Polar co-ordinates.

(d.) State Abel's test.

(e.) State Dirichlet's test.

(f.) Evaluate $\displaystyle\int_0^1 \int_0^\infty \frac{x}{1+y^2} \, dx \, dy$ i.e. $0 \le x \le 1$ and $0 \le y < \infty$.

(g.) Define line integral.

(h.) Define surface integral.

(I.) State Green's theorem in R^2.

(J.) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the straight line joining points $(0,0)$ and $(1,1)$ where $\vec{F} = x^2 \hat{i} - xy \hat{j}$

Q (2.) Evaluate $\iint_R x^2 \, dx \, dy$ where R is the circle $x^2 + y^2 = 1$ (5)

Group - B $4 \times 15 = 60$

Answer any four:-

Q3 (a.) Discuss the convergence of Beta function. $7\frac{1}{2}$

(b.) Discuss the convergence of Gamma function. $7\frac{1}{2}$

Q4.

(a.) Establish the relation between Beta and Gamma functions. $7\frac{1}{2}$

(b.) State and prove Duplication formula. $7\frac{1}{2}$

Q5. (a.) State and prove Dirichlet's theorem. $7\frac{1}{2}$

(b.) Solve :- $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$, $7\frac{1}{2}$

Q6 (a.) Show that $\iiint \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}} = \frac{\pi^2}{8}$ where $x^2+y^2+z^2 \le 1,\ x \geq 0,\ y \geq 0,\ z \geq 0.$ $7\frac{1}{2}$

(b.) Transform $\iiint v\,dx\,dy\,dz$ by the Polar transformations $x = r\sin\theta\cdot\sin\phi,\ y = r\sin\theta\cdot\cos\phi,\ z = r\cos\theta$ $7\frac{1}{2}$

Q7 (a.) State and prove Gauss's divergence theorem. $7\frac{1}{2}$

(b.) State and prove Stoke's theorem. $7\frac{1}{2}$

Q8 (a.) For a closed surface 'S' enclosing a volume 'V', show that $\iint_S \vec{r}\cdot\vec{n}\,ds = 3V$ $7\frac{1}{2}$

(b.) State and prove Green's theorem in R^2 $7\frac{1}{2}$

U.G. Core Course – C9 (SET – 1) Semester – IV

Subject – Mathematics

MECHANICS – I

F.M – 75

Group – A (Compulsory)

Q(1.) $1 \times 10 = 10$

(a.) Define coplanar forces.

(b.) Define Astatic Centre.

(c.) write the laws of friction.

(d.) Define angle of friction.

(e.) Define Cone of friction.

(f.) Define angular velocity and acceleration.

(g.) Define Simple Harmonic motion (S.H.M)

(h.) Define impulse and work.

(I.) Define Torque and angular momentum.

(J.) State Hooke's law.

Q(2.) Prove that angle of friction is equal to angle of repose. 5

Group – B $4 \times 15 = 60$

Answer any four:–

(3.)(a.) Find the equation of line of action of the resultant of a system of co-planar forces acting upon a rigid body. 7

(b.) A body is placed on a rough plane inclined to horizon at an angle greater than the angle of friction and supported by a force acting in a vertical through the line of greatest slope. Find the limits between which the force must lie.

8

(4.) (a.) Find the condition of equilibrium of a particle constrained to move on a rough curve under any given coplanar forces. (8)

(b.) Forces $P_1, P_2, P_3, P_4, P_5, P_6$ act along the sides of a regular hexagon taken in order. Show that if they are in equilibrium,

$$\sum_{r=1}^{6} P_r = 0 \text{ and } P_1 - P_4 = P_3 - P_6 = P_5 - P_2 \quad (7)$$

(5.) (a.) Find the tangential and normal acceleration of a particle moving in a plane curve. (8)

(b.) Derive expressions of radial and transverse accelerations. (7)

(6.) (a.) Prove that the work done against the tension in stretching a light elastic string is equal to the product of its extension and the mean of the initial and final tensions. (8)

(b.) A particle whose mass is 'm' is acted upon by a force $m\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin; if it starts from rest at a distance 'a'. Show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. (7)

(7.) (a.) If in a S.H.M, u, v, w be the velocities at distances a, b, c from a fixed point on the straight line which is not the centre of force; show that the periodic time T is given by the following equation.

$$4\pi^2 (b-c)(c-a)(a-b) = T^2 \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} \quad (8)$$

(7.) (b.) Find the extension of a heavy elastic string of weight W and natural length l hanged from one end and supporting a weight W_1 at the other end where λ is the modulus of elastility of the string. (7)

(8.)

(a.) A Particle starts from rest and moves along a straight line with an acceleration which is always directed towards a fixed Point and varies as the distance from the fixed Point. Prove that the time Period is independent of amplitude. (8)

(b.) If no resultant external force acts on a system of bodies, then the algebraic sum of momentum of the system in any direction is unaltered by the mutual action of the bodies, whatever the original condition of the system may be. (7)

U.G. Core Course - C 10 (Set - 1)

Semester - IV

Subject - Mathematics

F.M - 75

RING THEORY

Group - A (Compulsory)

$1 \times 10 = 10$

Q(1.) (a.) Define a commutative ring with unity.

(b.) Define an integral domainand a field.

(c.) Define characteristic of a ring.

(d.) Define simple ring.

(e.) Define a prime ideal.

(f.) Define a maximal ideal.

(g.) Define ring homomorphisms.

(h.) Define isomorphism in rings.

(I.) Define polynomial ring.

(J.) what is principal ideal.

Q(2.) Prove that every finite integral domain is a field. (5)

Group - B $4 \times 15 = 60$

Answer any four:-

Q(3.) (a.) Prove that every ideal is a subring but every subring is not necessarily an ideal. (7)

(b.) Give an example of a left ideal in a Ring which is not a right ideal and of a right ideal which is not a left ideal. (8)

(4.) (a.) A ring 'R' is without zero divisors iff the cancellation laws hold in 'R'. Prove it (8) (b.) State and prove the necessary and sufficient conditions for a non empty subset of a ring R to be a subring of R. (7)

(5.) (a.) For any two ideals A and B of a ring R, prove that $A \cup B$ is an ideal of R iff either $A \subseteq B$ or $B \subseteq A$. (8)

(b.) Show that in a commutative ring 'R' with identity, an ideal M is a maximal ideal iff R/M is a field. (7)

(6.) (a.) Prove that every homomorphic image of a ring is isomorphic to some quotient ring. (8)

(b.) Show that (i) $a+a=0$ (ii) $a+b=0 \Rightarrow a=b$ (iii) $ab=ba$ if it is given that $a^2=a \;\forall a$ in any ring R, $b \in R$. (7)

(7.) (a.) Prove that the ring $R[x]$ is not a field even if R is a field. (8)

(b.) Prove that the set of polynomials over a ring R is a ring w.r.t. addition and multiplication of polynomials. (7)

(8.) (a.) Prove that in an integral domain D with unity, every prime element is irreducible. (7)

(b.) Show that in a unique factorization domain every irreducible element is prime. (8)