U.G. Core Course – C 8 (SET 2)

ANALYSIS – II

Sem – IV

F.M – 75

GROUP–A (Compulsory)

Q (1.) $1 \times 10 = 10$

(a.) Write the value of $\Gamma(\frac{1}{2})$

(b.) Prove symmetric property for Beta function.

(c.) Check whether $\int_0^\infty \sin x \, dx$ is convergent or not.

(d.) Define improper integral of first kind and second kind.

(e.) State Liouville's extension of Dirichlet's theorem.

(f.) Define conservative vector field.

(g.) Define volume integral

(h.) If $\vec{F} = (3x^2+6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$ then evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is a straight line joining (0,0,0), (1,1,1).

(I.) What is the value of $\iint_S \vec{n} \cdot \text{curl}\vec{F} \, ds$ for a closed surface 'S' enclosing a volume 'V'.

(J.) Write the value of $\int_0^\infty e^{-x^2} dx$

Q (2.) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ of $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$ where C is the curve $y = x^3$ in xy plane from the point (1,1) to (2,8) (5)

GROUP–B

Answer any four:– $4 \times 15 = 60$

(3.) (a.) Examine the convergence of $\int_0^1 x^{n-1} \cdot \log x \, dx$ (8)

(b.) Change the order of integration in $\int_0^{2a}\int_{\frac{x^2}{4a}}^{3a-x} f(x,y)\,dx\,dy$ (7)

(4.) (a.) Evaluate $\iiint x^{l-1}\cdot y^{m-1}\cdot z^{n-1}\,dx\,dy\,dz$ where x, y, z are always positive but limited by the condition $\left(\frac{x}{a}\right)^p+\left(\frac{y}{b}\right)^q+\left(\frac{z}{c}\right)^r \le 1.$ (8)

(b.) Evaluate:- $\int_0^{\infty} e^{-\left(x^2+\frac{a^2}{x^2}\right)}\,dx$ (7)

(5.) (a.) Prove Liouville's extension of Dirichlet's theorem (8)

(b.) Evaluate $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^2}$, the integral being taken throughout the volume bounded by the planes $x=0,\ y=0,\ z=0,\ x+y+z=1.$ (7)

(6.) (a.) Change the order of integration in $\int_0^{a}\int_{\sqrt{ax-x^2}}^{\sqrt{ax}} f(x,y)\,dx\,dy$ (8)

(b.) If $\vec{F}=2xy\,\hat{i}-yz\,\hat{j}+x^2\hat{k}$ then evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$ where S is cube bounded by the co-ordinate planes and the planes $x=a,\ y=a,\ z=a.$ (7)

(7.) (a.) Check the convergence of improper integral $\int_a^b \frac{dx}{(x-a)^n}$ (8)

(b.) Verify Stoke's theorem for $\vec{F}=(3x-y)\,\hat{i}-yz^2\hat{j}-y^2z\,\hat{k}$ where S is the upper half of $x^2+y^2+z^2=1$ and C is its boundary. (7)

(8.) (a.) Examine the convergence of $\int_0^1 \frac{dx}{x^{1/2}\cdot(1-x)^{1/3}}$ (8)

(b.) Evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$ where $\vec{F}=4xz\,\hat{i}-y^2\hat{j}+yz\,\hat{k}$ and S is the surface bounded by unit cube. (7)

U.G. Core Course – C9 (Set 2) Sem IV

MECHANICS–I F.M. 75

Group–A (Compulsory)

Q (1.) $1 \times 10 = 10$

(a.) Define static equilibrium.

(b.) [redacted], write differential equation of S.H.M.

(c.) Define conservative forces.

(d.) Define non-conservative forces.

(e.) Define simple pendulum.

(f.) A point moves in a straight line so that after time 't', the distances 's' from a fixed point O on the line is given by $s = (t-1)^2 \cdot (t-3)$. Find the time when velocity is zero.

(g.) If μ is the coefficient of friction and 'λ' is the angle of friction then $\mu = \ldots\ldots\ldots$

(h.) Define radial and transverse velocities of a particle.

(I.) Write the necessary and sufficient conditions for equilibrium of a system of co-planar forces.

(J.) The least force necessary to move a weight W along a rough horizontal plane is ……………

(i) $W \tan\lambda$ (ii) $W \sin\lambda$ (iii) W (iv) $W \cdot \cot\lambda$ where λ is the angle of friction.

Q (2.) Show that impulse of a force in a given time is equal to change in momentum in the direction of the force during that time. (5)

Group – B

Answer any four:– $4 \times 15 = 60$

Q (3.) (a.) Prove that in all cases, a given system of co-planar forces not in equilibrium can be changed by two forces, one of which acts through a given point and the other along a straight line. (7)

(b.) Forces P, Q, R act along the lines $x=0$, $y=0$ and $x\cos\alpha + y\sin\alpha = p$. Find the magnitude of the resultant and the equation of its line of action. (8)

(4.)

(a.) Derive general conditions of equilibrium of a rigid body under co-planar forces. (7)

(b.) A solid hemisphere of weight 'w' rests in limiting equilibrium with its curved surface on a rough inclined plane and its plane face is kept horizontally by a weight 'p' attached to a point on its rim. Find the coefficient of friction. (8)

(5.)(a.) A uniform rod rests in vertical plane within a fixed hemispherical bowl whose radius is equal to the length of the rod. If μ is the coefficient of friction between the bowl and the rod then in limiting equilibrium, find the inclination of the rod to the horizon. ($7\frac{1}{2}$)

(b.) A sphere whose radius is 'a' and whose centre of gravity is at a distance 'c' from the centre, rests in limiting equilibrium on a rough plane inclined at an angle 'α' to the horizon. Show that it may be turned through an angle $2\cos^{-1}\left(\frac{a\sin\alpha}{c}\right)$ and still be in equilibrium. ($7\frac{1}{2}$)

(6.)(a.) Derive the expressions for radial and transverse velocities of a particle in plane curvilinear motion in terms of its polar co-ordinates. (8)

(b.) Derive expressions for tangential and normal velocities. (7)

(7.) (a.) A Particle starts with a given velocity 'V' and moves under a retardation equal to 'K' times the space described. Show that the distance traversed before it comes to rest is $\frac{V}{\sqrt{K}}$. (8)

(b.) Prove that if a Particle moves under the action of a Conservative system of forces, the sum of its Kinetic and Potential energies is constant throughout the motion. (7)

(8.) (a.) A light elastic string of natural length 'a' and modulus of elasticity 'λ' is suspended by one end, to the other end is tied a particle of weight 'mg'; the Particle is slightly pulled down and released. Discuss the motion. (8)

(b.) An insect crawls at a constant rate 'u' along the spoke of a cart wheel of radius 'a', the cart is moving with velocity 'v'. Find the acceleration along and perpendicular to the spoke. (7)

$\boxed{\text{or}}$

A bullet of mass 'm' moving with a velocity 'u' strikes a mass M, which is free to move in the direction of the bullet and is embedded to it. Find the loss of kinetic energy.

U.G. Core Course – C 10 (Set – 2) Sem – IV

RING THEORY F.M. – 75

Group – A (Compulsory)

Q(1.) $1 \times 10 = 10$

(a.) For all 'a' in a Ring 'R', Prove that $a \cdot 0 = 0 \cdot a = 0$

(b.) Define a division ring or skew field.

(c.) Define a subring.

(d.) Define an ideal.

(e.) Define Kernel of ring homomorphism.

(f.) Define an irreducible element of a commutative ring with unity.

(g.) Define zero divisors.

(h.) Define quotient rings.

(I.) what do you understand by sum and Product of Polynomials.

(J.) Let $f: R \to R'$ is a ring homomorphism. Then Prove that $f(-a) = -f(a) \ \forall \ a \in R$

Q 2 Prove that every field is always an integral domain but its converse is not necessarily always true. 5

Group – B

Answer any four :– $4 \times 15 = 60$

Q 3 (a.) Let $f: R \to R'$ be a ring homomorphism and A be any ideal of ring R. Then Prove that $f(A)$ is an ideal of $f(R)$. 8

(b.) Let A and B are any two ideals of a Ring 'R'. Then Prove that

(i) $A \cap B$ is also an ideal of R 7

(ii) $A + B$ is also an ideal of R

(4.)(a.) Let R be a ring and S be any ideal of R.
Let $f : R \to R/S$ is defined by $f(a) = S + a \; \forall a \in R$.
Then show that 'f' is a homomorphism of 'R' onto 'R/S'. (8)

(b) Show that in a commutative ring 'R' with identity, an ideal 'P' is a prime ideal iff R/P is an integral domain. (7)

(5.)(a.) If R is a commutative ring then prove that $R[x]$ is a commutative ring. (7)

(b.) Prove that a ring without unity can be embedded in a ring with unity. (8)

(6.)(a.) Let $f : R \to R'$ is a ring homomorphism. Then, prove that Kernel of 'f' is an ideal of ring R. (7)

(b.) Show that the set $S = \{a + b\sqrt{2}, a, b \in Q\}$ form a field w.r.t. ordinary addition and multiplication. (8)

(7.)(a.) State and prove second isomorphism theorem ($7\frac{1}{2}$)

(b.) State and prove third isomorphism theorem. ($7\frac{1}{2}$)

(8.)(a.) Explain the following, with examples. (8)

(i) Eisenstein's criterion
(ii) Principal ideal domains

(b.) Prove that if a Ring 'R' has no proper zero divisors then the polynomial ring $R[x]$ has also no proper zero divisors (7)