U.G. Core Course – C3 (SET 2) Sem – II

ANALYSIS – I F.M. – 45

Group–A (Compulsory)

1×10 = 10

Q(1.) (a.) Define least upper bound of a sequence.

(b.) Define greatest lower bound of a sequence.

(c.) Define derived set of a set.

(d.) Write the derived set of $\{1, \frac{1}{2}, \frac{1}{3}, \cdots\}$

(e.) Define a monotonic increasing sequence.

(f.) Define limit of a sequence.

(g.) State D'Alembert's ratio test.

(h.) State Leibnitz test for alternating series.

(I.) State De Morgan's and Bertrand's test.

(J.) Write the set of limits points of $]2,3[$.

Q(2.) Prove that the limit of a convergent sequence is always unique. (5)

Group–B

Answer any four questions:–

Q(3.) (a.) Prove that the set of rational numbers is not order-complete. (8)

(b.) Show that $\sqrt{2}$ is not a rational number. (7)

Q(4.) (a.) Define the following:– (i) Deleted neighbourhood of a point (ii) interior point of a set (iii) adherent point of a set (iv.) closure of a set. (8)

(b.) Prove that union of finite number of closed sets is always a closed set but union of infinite number of closed sets is not necessarily closed. (7)

Q(5.) (a.) State and Prove Cauchy's first theorem on limits. (8)

(b.) Prove that the sequence $\{a_n\}$ defined by

$$a_{n+1} = \sqrt{7+a_n}\,, \quad a_1 = \sqrt{7}$$ (7)

converges to the positive root of $x^2 - x - 7 = 0$

Q(6.) (a.) If $\{a_n\}$ be a sequence of positive real numbers such that $a_n = \frac{1}{2}(a_{n-1} + a_{n-2}) \;\forall\; n > 2$ (8)

then show that $\{a_n\}$ converges. Also find $\lim\limits_{n\to\infty} a_n$.

(b.) If $\{a_n\}$ be a sequence of positive real numbers such that

$$a_n = \sqrt{a_{n-1} \cdot a_{n-2}}\;;\; n > 2$$ (7)

then show that the sequence converges to $(a_1 a_2^2)^{1/3}$

Q(7.) (a.) Test the convergence of the series $\sum u_n$ where

$$u_n = \sqrt[3]{n^3+1} - n$$ (8)

(b.) Test the convergence of the series $\sum u_n$ where

$$u_n = \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}} = \frac{1}{\left(1+\frac{1}{\sqrt{n}}\right)^{n^{3/2}}}$$ (7)

Q(8.) (a.) Test the convergence of the series

$$1 + \frac{x}{\lfloor 1} + \frac{2^2 \cdot x^2}{\lfloor 2} + \frac{3^3 \cdot x^3}{\lfloor 3} + \cdots \text{ for } x > 0$$ (8)

(b.) Test the convergence of the series

$$1 + \frac{\alpha \cdot \beta}{1 \cdot \gamma} \cdot x + \frac{\alpha \cdot (\alpha+1) \cdot \beta \cdot (\beta+1)}{1 \cdot 2 \cdot \gamma \cdot (\gamma+1)} x^2 + \frac{\alpha \cdot (\alpha+1)(\alpha+2) \cdot \beta \cdot (\beta+1) \cdot (\beta+2)}{1 \cdot 2 \cdot 3 \cdot \gamma(\gamma+1)(\gamma+2)} \cdot x^3 + \cdots\cdots$$

for all positive values of x; α, β, γ being all positive. (7)

U.G. Core Course – C 4 (SET 2) SEM – II

Integral Calculus and Analytical Geometry 3D F.M. 75

Group – A (Compulsory)

$1 \times 10 = 10$

Q ① (a.) Evaluate:- $\int \frac{1}{x^2+4x+5} dx$

(b.) Evaluate:- $\int \frac{1}{e^x-1} dx$

(c.) Prove that $\int_a^b f(x) dx = \int_a^b f(t) dt$

(d.) Write the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(e.) Write the reduction formula for $\int \tan^n x \, dx$

(f.) Write the direction cosines of x, y, z axes

(g.) Write the condition for two lines to be perpendicular.

(h.) Write the intercepts made on the axes by the plane $7x+3y-11z+29=0$

(I.) Define orthogonal spheres.

(J.) Find equation of sphere with centre $(2,-1,3)$ and radius 4.

Q ② Integrate $\int \sqrt{\tan x} \, dx$ (5)

Group – B

Answer any four:- $4 \times 15 = 60$

Q ③ (a.) Evaluate $\int_0^\infty \frac{\sin x}{x} dx$ (8)

(b.) Evaluate $\int_0^{\pi/2} \log \sin x \, dx$ (7)

(4.) (a.) Find reduction formula for $\int \cos^m x \cdot \cos nx \, dx$ (8)

(b.) Find the whole length of astroid $x^{2/3} + y^{2/3} = a^{2/3}$ (7)

(5.) (a.) Find the whole length of Cardioid $r = a(1+\cos\theta)$ (8)

(b.) Find the volume and surface area of the solid obtained by revolving $r = a(1+\cos\theta)$ about its initial line. (7)

(6.) (a.) Show that the lines whose direction cosines are given by $l+m+n=0$ and $2mn+3nl-5lm=0$ are perpendicular to each other. ($7\frac{1}{2}$)

(b.) Find the angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ and $l^2+m^2-n^2=0$ ($7\frac{1}{2}$)

(7.) (a.) If a line makes angles $\alpha, \beta, \gamma, \delta$ with the diagonals of a cube, then prove that

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = \frac{8}{3}$$ (8)

(b.) Derive the equation of the plane in normal form. (7)

(8.) (a.) Find the equation of the plane through the line of intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$ and perpendicular to the plane $5x+3y+6z+8=0$ (7)

(b.) Two spheres of radii 'r_1' and 'r_2' cut orthogonally. Prove that the radius of common circle is

$$\frac{r_1 \cdot r_2}{\sqrt{r_1^2+r_2^2}}$$ (8)