U.G. Core Course – C 13 (Set 2) Semester VI

MECHANICS – II Mathematics

F.M. 75

Group – A (Compulsory)

Q(1.) $1 \times 10 = 10$

(a.) Write the Cartesian equation of the common catenary.

(b.) Define conservative field of force.

(c.) Write the forces which must appear in the equation of virtual work.

(d.) State the general conditions of equilibrium of any given system of forces acting at given points of a rigid body.

(e.) What are the invariants when a system of forces acts on a rigid body?

(f.) Write the expression for range of a projectile.

(g.) What is compound pendulum.

(h.) State Newton's law of Gravitation.

(I.) What are perihelion and aphelion in planetary motion.

(J.) Find minimum time of oscillation of a compound pendulum.

Q(2) List the forces which may be ommitted in forming the equation of virtual work. 5

Group – B

Answer any four:– $4 \times 15 = 60$

(3.)(a.) A string of length 'a' forms the shorter diagonal of a rhombus formed of four uniform rods, each of length 'b' and weight 'w', which are hinged together. If one of the rods be supported in a horizontal position, prove that the tension of the string is 7

$$\frac{2w(2b^2-a^2)}{b\sqrt{4b^2-a^2}}$$

(3.) (b.) A heavy chain of length $2l$ has one end tied at 'A' and the other is attached to a small heavy ring which can slide on a rough horizontal rod which passes through A. If the weight of the ring be 'n' times the weight of the chain, show that the greatest possible distance from A is $\frac{2l}{\lambda}\cdot\log\left[\lambda+\sqrt{1+\lambda^2}\right]$ where $\frac{1}{\lambda} = \mu\cdot(2n+1)$ and μ is coefficient of friction. (8)

(4.) (a.) A solid homogeneous hemisphere of radius 'a' has a solid right cone of the same substance constructed on its base; the hemisphere rests on the convex side of a fixed sphere of radius R, the axis of the cone being vertical. Show that the greatest height of the cone consistent with stability for a small rolling displacement is $\frac{a}{R+a}\left[\sqrt{(3R+a)\cdot(R-a)} - 2a\right]$ (8)

(b.) A single force is equivalent to component forces X, Y, Z along co-ordinate axes and to couples L, M, N about these axes; show that the magnitude of the single force is $\sqrt{X^2+Y^2+Z^2}$ and the equation to its line of action is $\frac{yZ - zY}{L} = \frac{zX - xZ}{M} = \frac{xY - yX}{N} = 1$. (7)

(5.) (a.) A force 'P' acts along the axis of x and another force $n\cdot P$ acts along a generator of the cylinder $x^2+y^2=a^2$; show that the central axis lies on the cylinder $n^2\cdot(nx-z)^2+(1+n^2)^2y^2 = n^4\cdot a^2$ (7)

(b.) Find the equation of trajectory of a projectile thrown with a velocity 'u' making an angle 'α' with horizontal. (8)

(6.) (a.) Find the time of description of a given arc of an elliptic orbit starting from the nearer end of major axis. (7)

(b.) A planet is describing an ellipse about the Sun as focus. Show that its velocity away from the sun is greatest when the radius vector to the planet is at right angles to the major axis of the path. (8)

(7.) (a.) A solid homogeneous cone of height 'h' and vertical angle '2α' oscillates about a horizontal axis through its vertex. Show that the length of simple equivalent pendulum is $\frac{h}{5}(4+\tan^2\alpha)$ (8)

(b.) Deduce the general equations of motion of a rigid body from D'Alembert's Principle. (7)

(8.) (a.) Show that the centres of oscillation and suspension in the case of a compound pendulum are interchangable. (8)

(b.) Find the time of description of a given arc of parabolic orbit of a comet starting from the perihelion. (7)

U.G. Core Course – C 14 (SET 2) Semester – VI

Numerical Analysis F.M.:– 75

Group – A (Compulsory)

1×10 = 10

Q(1.) (a.) Prove that $E = \Delta + 1$ where E is shift operator and Δ is forward difference operator.

(b.) Prove that $\nabla \cdot E = \Delta$ where E is shift operator and Δ, ∇ are forward and backward difference operators respectively.

(c.) Write Newton's forward interpolation formula.

(d.) Write trapezoidal rule for numerical integration.

(e.) Find the smallest integral interval in which a real root of the equation $x \cdot \log_{10} x = 1.2$ exists.

(f.) Distinguish between interpolation and extrapolation.

(g.) Find the function whose first difference is $x^3 + 2x^2 + 5x + 12$.

(h.) Find ~~the~~ one interval in which a real root of $x^3 - x - 1 = 0$ lies.

(I.) Write geometrical interpretation of Newton-Raphson's method.

(J.) State general quadrature formula for equidistant ordinates.

Q(2) Find a root of the equation $x^3 - 4x - 9 = 0$ using bisection (Bolzano) method in four stages. (5)

Group – B

4×15 = 60

Answer any four:–

(3.) (a.) Find a real root of $x^3 - x - 1 = 0$ by Newton-Raphson's method. (8)

(b.) Prove that bisection method always converges. (7)

(4.)(a.) State and prove the fundamental theorem of differential calculus. (8)

(b.) Derive Newton's interpolation formula for equal intervals for the polynomial $f(x)$ (7)

(5.)(a.) obtain the missing term in the following table:- (7)

X	0	1	2	3	4
Y	1	3	9	—	81

(b.) Estimate the population in '1905' from following data: (8)

Year	1891	1901	1911	1921	1931
population	98752	132285	168076	195690	246050

(6.)(a.) Derive Simpson's one third rule on numerical integration (8)

(b.) Find $f(x)$ from following table:- (using Newton's divided difference formula) (7)

x	1	2	4	7	12
y	22	30	82	106	216

(7.)(a.) A second degree polynomial passes through (0,1), (1,3), (2,7) and (3,13). Find the polynomial (7)

(b.) Represent the function $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ into factorial notation. (8)

(8.)(a.) Compute $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\,dx$ using Simpson's $\frac{1}{3}$rd and $\frac{3}{8}$th rule. (8)

(b.) Using Picard's method, solve $\frac{dy}{dx} = x - y$; $y(0) = 1$ upto third approximation. (7)

U.G. Core Course – DSE 3 (Set 2) Sem – VI

Linear Programming F.M. 75

Group – A (Compulsory)

Q(1.) $1 \times 10 = 10$

(a.) Define extreme point of a convex set.

(b.) Define convex Polyhedron.

(c.) Define a hyperplane.

(d.) Define a hypersphere.

(e.) Define optimal Basic feasible solution in a L.P.P.

(f.) Define artificial variables in a L.P.P.

(g.) Write the two methods involved in solving a L.P.P. which contain constraints with '$\geq$' or '$=$' signs.

(h.) What are the three methods to solve a transportation problem.

(I.) When a transportation problem is called unbalanced?

(J.) Is $x_1 = 5$, $x_2 = 0$, $x_3 = -1$ a basic feasible solution of equations $x_1 + 2x_2 + x_3 = 4$; $2x_1 + x_2 + 5x_3 = 5$? Write 'Yes' or 'no'?

Q (2) Show by an example that union of two convex sets is not necessarily a convex set. (5)

Group – B

Answer any four:– $4 \times 15 = 60$

Q (3.) (a.) Show that the set $S = \{(x,y) : 3x^2 + 2y^2 \leq 6\}$ is convex. (8)

(b.) Show that the set $S = \{(x,y) : xy \leq 1, x \geq 0, y \geq 0\}$ is not convex. (7)

Q (4.) (a.) Use simplex method to solve the L.P.P.

$$\text{Max. } Z = 4x_1 + 5x_2$$

Subject to $2x_1 + 3x_2 \leq 12$

$3x_1 + x_2 \leq 8$; $x_1 \geq 0$; $x_2 \geq 0$ (8)

(b.) Write the dual of $\text{Max } Z = 3x_1 + 4x_2$ subject to $4x_1 + 2x_2 \leq 7$; $x_1 + 3x_2 \leq 5$; x_1, x_2 unrestricted (7)

(5.) (a.) Solve by two Phase method:- $\text{Max } z = 3x_1 - x_2$ subject to $2x_1 + x_2 \geq 2$; $x_1 + 3x_2 \leq 3$; $x_2 \leq 4$; $x_1, x_2 \geq 0$. (8)

(b.) Solve graphically:- $\text{Max } z = 5x_1 + 3x_2$ subject to $3x_1 + 5x_2 \leq 15$; $5x_1 + 2x_2 \leq 10$; $x_1, x_2 \geq 0$ (7)

(6.) (a.) Solve:- $\text{Max } z = 4x_1 + 3x_2$ subject to $x_1 + x_2 \leq 5$; $x_1 + 2x_2 \geq 8$; $3x_1 + 2x_2 \geq 14$; $x_1, x_2 \geq 0$ (8)

(b.) use duality to solve $\text{Min. } z = 3y_1 + y_2$ subject to $2y_1 + 3y_2 \geq 2$; $y_1 + y_2 \geq 1$; $y_1, y_2 \geq 0$ (7)

(7.) (a.) Find all basic solutions to the following system of equations: $x_1 + 2x_2 + x_3 = 4$; $2x_1 + x_2 + 5x_3 = 5$ (8)

(b.) Solve the following transportation problem: (7)

	D_1	D_2	D_3	D_4	D_5	D_6	$a_i \downarrow$
S_1	10	12	13	8	14	19	18
S_2	15	18	12	16	19	20	22
S_3	17	16	13	14	10	18	39
S_4	19	18	20	21	12	13	14
$b_j \rightarrow$	10	11	13	20	24	15	

(8.) (a.) Solve the following transportation problem:- (8)

	D_1	D_2	D_3	D_4	$a_i \downarrow$
S_1	25	17	25	14	30
S_2	15	10	18	24	50
S_3	16	20	8	13	60
$b_j \rightarrow$	30	30	50	50	

(b.) Solve the following assignment problem:- (7)

	I	II	III	IV
A	10	12	9	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

U.G. Core Course – DSE 4 (Set 2) Sem VI

FLUID MECHANICS – I F.m:- 75

Group – A (Compulsory)

$1 \times 10 = 10$

Q(1.) (a.) Write the pressure equation of a liquid at rest under gravity.

(b.) Write differential equation of a streamline.

(c.) Define Centre of gravity.

(d.) Write any one application of Archimedes' principle.

(e.) Define buoyancy.

(f.) Define conservative field of force.

(g.) Write down Euler's dynamical equations of motion in Cartesian form.

(h.) Write expanded form of Lagrange's Hydrodynamical Equations.

(I.) Define an incompressible fluid flow.

(J.) What is conservation of mass.

Q(2) Find centre of pressure of rectangle immersed in a homogeneous liquid with one side on surface. (5)

Group – B

Answer any four :- $4 \times 15 = 60$

Q(3) (a.) A fine tube bent in the form of an ellipse is held with its major axis vertical and is filled with their liquids whose densities are ρ_1, ρ_2, ρ_3 taken in order round the elliptic tube. If r_1, r_2, r_3 be the distances of point of separation from either foci, prove that
$$r_1(\rho_1-\rho_2)+r_2(\rho_2-\rho_3)+r_3(\rho_3-\rho_1)=0. \quad (8)$$

(b.) If α, β, γ be the depth of vertices A, B, C of a triangle ABC respectively, find the depth of its centre of pressure. (7)

(4.) (a.) A closed tube in the form of an equilateral triangle contains equal volumes of three liquids which do not mix and is placed with its lowest side horizontal. Prove that if the densities of the liquids are in A.P., their surface of separation will be at points of trisection of the sides of the triangle. (8)

(b.) Prove that the surface of separation of two liquids of different densities which do not mix at rest under gravity is a horizontal plane. (7)

(5.) (a.) (i) State conditions for equilibrium of a freely floating body.

(ii) Prove that the depth of the centre of pressure always exceeds that of the centre of gravity of a plane area. (8)

(b.) A circular tube is half full of liquid and is made to revolve round a vertical tangent line with angular velocity ω. If 'a' be the radius of the tube, prove that the diameter passing through the free surfaces of the liquid is inclined at an angle $\tan^{-1}\left(\frac{\omega^2 a}{g}\right)$ to the horizon. (7)

(6.) (a.) Derive the equation of continuity in cylindrical polar co-ordinates. (8)

(b.) Establish the equation of continuity obtained by Euler's method. (7)

(7.) (a.) A mass of fluid is in motion so that the lines of motion lie on the surface of coaxial cylinders. Show that the equation of continuity is $\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u) + \frac{\partial}{\partial z}(\rho v) = 0$ (8)

(b.) The velocity 'q' in a three dimensional flow field for an incompressible fluid is given by $q = 2x\hat{i} - y\hat{j} - z\hat{k}$. Is it a possible field? Determine the equation of the streamline passing through the point $(1,1,1)$ (7)

(8.) A triangle ABC is immersed in a liquid with the vertex C in the surface and the sides AC and BC equally inclined to the surface. Show that the vertical through C divides the triangle into two parts, the fluid pressures on which are as $(b^3 + 3ab^2) : (a^3 + 3a^2b)$ (15)