P.G. Core Course – CCMAT203 M.Sc. Semester – 2

Differential Geometry and Tensor Analysis Mathematics

F.M – 70

Group A (Compulsory)

Q(1.) (a.) Define curvature and torsion. $1 \times 5 = 5$

(b.) Define the edge of regression.

(c.) Define Parametric curves.

(d.) State Serret-Frenet formula.

(e.) Define a Tensor.

Q(2.) Deduce the formula $ds^2 = E\,du^2 + 2F\,du\,dv + G\,dv^2$ (5)

Group – B

Answer any four :– $4 \times 15 = 60$

Q(3.) (a.) Deduce formula for curvature of normal section K_n in terms of fundamental magnitudes. $7\frac{1}{2}$

(b.) Calculate the fundamental magnitude for the right helicoid given by $x = u\cos\phi$, $y = u\sin\phi$, $z = c\phi$ $7\frac{1}{2}$

Q(4.) (a.) Explain the notion of lines of curvature and obtain its differential equation $7\frac{1}{2}$

(b.) Explain the concept of conjugate directions and find an analytical expression for two directions to be conjugate. $7\frac{1}{2}$

Q(5.) (a.) Show that the curve $u + v =$ constant are geodesics on a surface with metric $(1+u^2)\,du^2 - 2uv\,du\,dv + (1+v^2)\,dv^2$ $7\frac{1}{2}$

(b.) Show that the necessary and sufficient condition that a curve on a developable surface be a geodesic is that the surface be the rectifying developable of the curves. $7\frac{1}{2}$

(6.) (a.) If a geodesic on a surface of revolution cuts the meridian at any point at an angle ϕ, $u\sin\phi$ is constant, where u is the distance of the point from the axis. Prove it. 8

(b.) Explain the following:-

(i) Circular helix (ii) osculating sphere 7

(7.) (a.) Explain in brief :-

(i) Bertrand curves (ii) Principal directions and Principal curvatures. 8

(b.) Explain the following:-

(i) Tensor algebra and contraction. 7

(ii) Metric Tensor

(8.) State and prove quotient theorem for Tensors. 15

P.G. Core Course — CCMAT 202

M.Sc. Semester - 2

Mathematics

Ordinary Differential equations

F.M - 70

Group A (Compulsory)

$1 \times 5 = 5$

Q (1.) (a.) Write Lipschitz Condition.

(b.) Define Wronskian of Second order ODE.

(c.) Define eigen values.

(d.) Define eigen vectors.

(e.) Define Green's function.

Q (2.) Explain Picard's method of successive approximation for solving the differential equation (5)

$\frac{dy}{dx} = f(x,y)$; $y = y_0$ when $x = x_0$.

Group - B

Answer any four :- $4 \times 15 = 60$

Q (3.) (a.) State and Prove Picard's existence theorem. $7\frac{1}{2}$

(b.) Find Lipschitz Constant for the function $f(x,y) = x \cdot y^2$ in $D = \{(x,y) : |x| \leq 1, |y| \leq 1\}$ $7\frac{1}{2}$

Q (4.) (a.) Explain the following:-

Wronskian of 'n' functions and its Properties. $7\frac{1}{2}$

(b.) Explain ~~the~~ briefly:-

Existence ~~of~~ and Uniqueness of linear System of ODEs $7\frac{1}{2}$

Q (5.) Explain Annihilator method to solve non homogeneous ODE with Constant Coefficients by taking a suitable example. (15)

Q(6.) Using the definition of Green's function, Construct Green's function for the boundary-Value Problem (15)

$$\frac{d^2y}{dx^2} - y = x \; ; \; y(0) = y(1) = 0$$

Q(7.) Using Green's function, Solve the boundary-value Problem

$$-\left(\frac{d^2y}{dx^2} + y\right) = x \; ; \; y(0) = y\left(\frac{\pi}{2}\right) = 0$$

(15)

Q(8.) Explain Strum-Liouville boundary value Problem with homogeneous boundary Conditions by taking a Suitable example. (15)